SOME REMARKS ON THE STRUCTURE OF A NORMAL MAGNETO-HYDRODYNAMIC SHOCK WAVE

(ZAMECHANIA OTNOSITEL'NO STRUKTURY PERPENDIKULARNOI MAGNITO-GIDRODINAMICHESKOI UDARNOI VOLNY)

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The structure of normal hydrodynamic shock waves in a viscous, heat-conducting gas is dealt with in reference [1]. It is also interesting to study the limiting cases when one or two of the dissipative coefficients are small enough to be neglected. The structure of a shock wave in the absence of thermal conductivity was dealt with in another work [2] and in the absence of both conductivity and of viscosity in [3]. In this

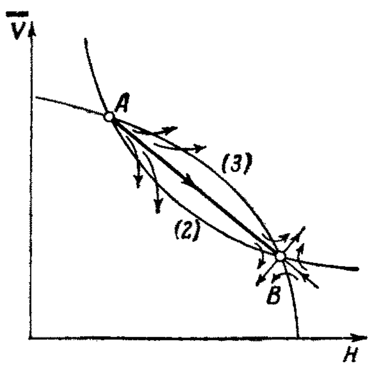

Fig. 1.

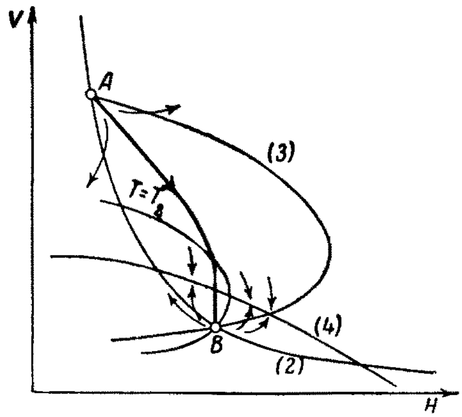

Fig. 2.

article we are dealing with the structure of a normal shock wave, taking into account the effects of thermal and electrical conductivity, but not of viscosity.

The system of equations describing the one-dimensional transient motion of a thermally and electrically conductive gas can be written down as

$$\nu_m \frac{dH}{dx} = vH - c_1$$

$$k\frac{dT}{dx}=-\frac{\gamma+1}{2(\gamma-1)}jv^2+\frac{\gamma}{\gamma-1}\left(J-\frac{H^2}{8\pi}\right)v-\left(\varepsilon-\frac{c_1}{4\pi}H\right)=f(vH) \qquad (1)$$

$$RT=j^{-1}\left[\left(J-\frac{H^2}{8\pi}\right)v-jv^2\right]$$

Here j is the mass flow, J is the momentum flow, ϵ is the energy flow, $c_1 = cE$.

A solution of this system, describing the structure of a shock wave, should depict a forward or progressive stream for $x = +\infty$. Therefore, to flows corresponding to $x = \pm\infty$ there should correspond points on the vH-plane, where the derivatives dT/dx and dH/dx become zero. These points, singular points of system (1), lie at the intersection of the hyperbola

$$Hv = c_1 \qquad (2)$$

and the curve

$$f(vH) = 0 \qquad (3)$$

If we eliminate v from equations (2) and (3) we arrive at a third degree equation for finding H. When $\gamma < 2$, one of its roots is negative and it lies, therefore within the range $v < 0$. When $\gamma > 2$, the greater root aways exceeds $\sqrt{8\pi J}$ and lies within the range $T < 0$. Therefore, within the range which interests us, namely $v > 0$, $T > 0$, there exist no more than two singularities of system (1). We will denote the point corresponding to the larger of the values of v as A and all quantities associated with this will carry the suffix 1. The other singular point will be referred to as B, and corresponding quantities will carry suffix 2. The magneto-hydrodynamic shock wave is represented by displacement from point A to point B.

It is possible to demonstrate that there exist only two relative locations for curves (2) and (3), as shown in Figs. 1 and 2. If we follow the values of the derivative dH/dx along the line $T =$ const and $f = 0$, we can see that in moving along line $f = 0$ in the vH-plane from point A to point B in the region $T > 0$, $Hv > c_1$, we find that the temperature increases. If we move between these singular points along the line $Hv = c_1$, the temperature either increases all the time, or rises at first and then drops. Integral curves can only emerge from a region bounded by curves (2) and (3) and $T = T_2$, so that point A is a nodal point and integral curves emerge from this point when x increases.

If, at point B, the inequality

$$jv > \frac{1}{2}\left(J-\frac{H^2}{8\pi}\right) \qquad (4)$$

is valid, this point is a "saddle" point (the location of the curves (2) and (3) can, then, be as shown in Figs. 1 or 2). In this case there exists one and only one integral curve passing from A to B and representing the structure of the shock wave (Fig. 1). This curve passes through the region bounded by curve (2), (3) and $H = H_2$

If at point B the opposite inequality to (4) holds, then point B becomes a nodal point from which integral curves energe for increasing values of x; (the location of the curves (2) and (3) for this case can then only be as in Fig. 2).

Line

$$jv = \frac{1}{2}\left(J - \frac{H^2}{8\pi}\right) \tag{5}$$

is a limiting line to which the integral curves approach from both sides. Because a continuous transition through this line is impossible, it can only be traversed as a jump in which T and H are discontinuous and V vanishes. In this latter case, the solution, which represents a shock wave, consists of the intersection of the integration curve going from point A to $H = H_2$, $T = T_2$, lying outside point (5) and the isothermal, isomagnetic shock or jump from this point to point B (Fig. 2).

Now let us deal with the case where the electrical conductivity of the gas is very large, so that

$$\varkappa = k / c_p \rho \gg \nu_m \tag{6}$$

Here the integral curve representing the shock wave goes from point A, along hyperbola (2), to point $T = T_2$. If this point is not point B, then further motion is along the line $T = T_2$ as far as point $H = H_2$. And if, in turn, this point is not point B, an isothermal, isomagnetic shock or jump will take place.

The width of the flow region represented by the intersection of the integral curve between point A and $T = T_2$ is determined by the thermal conductivity. The width of the rest of the flow region is determined by the magnetic viscosity, and, because of inequality (4), is considerably narrower than the previous portion. The flow region represented by the intersection of the integral curve, extending from the intersection of curves (2) and $T = T_2$, to point B, on increase in the inequality (6), tends to an isothermal (but not an isomagnetic) shock or jump.

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